Bourbaki and Algebraic Topology

by John McCleary

The principal aim of the Bourbaki group (L'Association des Collaborateurs de Nicolas Bourbaki) is to provide a solid foundation for the whole body of modern mathematics. The method of exposition is axiomatic and abstract, logically coherent and rigorous, proceeding normally from the general to the particular, a style found to be not altogether congenial to many readers. The ongoing series of books began with Éléments de Mathématiques in 1939, and other books on algebra, set theory, topology, and other topics have followed. Many books in the series have become standard references, though some mathematicians are critical of their austerely abstract point of view.

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It is now more than 70 years ago that the founders of *Le Comité de rédaction du traité d'analyse* met in Paris at the *Café A. Capoulade*, 63 boulevard Saint-Michel, to discuss the drafting of a textbook on analysis. This meeting included (recent centenarian) Henri Cartan (1904–), Claude Chevalley (1909–1984), Jean Delsarte (1903–1968), Jean Dieudonné (1906–1992), René de Possel (1905–1974), and André Weil (1906–1998). The fate of this project is the story of the *Bourbaki*, or should I say, the story of the character Nicolas Bourbaki, author of *Éléments de mathématique*, a series of influential expositions of the basic notions of modern mathematics.

This talk is based on a wild goose chase after a document. The project was supported by the Gabriel Snyder Beck Fund at Vassar College that funds research on anything French. In early 2000 I learned at a meeting in Oberwohlfach that an archive of papers and internal documents of the Bourbaki was soon to be opened to scholars in Paris. The Beck fund provided me the means to visit the archive. The managers of this archive, Liliane Beaulieu and Christian Houzel, showed me great hospitality during my visit to Paris in July 2003, and made it possible for me to rummage through the Bourbaki papers.

Historical research poses questions, to which various methods may be applied. My interests include the history of algebraic topology, a subject whose development during the twentieth century influenced a great deal of that century's mathematics. The years following the Second World War represent a high point in this story, and several important members of Bourbaki contributed to this development. However, algebraic topology does not appear among the topics treated in *Éléments*—admittedly many other important topics were also omitted. The involvement of so many pioneering topologists makes this omission stand out.

While a graduate student, I collected a rumor that there was a manuscript, 200 pages long, prepared for *Élements* by Cartan, Koszul, Eilenberg, and Chevalley, treating algebraic topology. Furthermore, this document was based on the use of differential forms, that is, algebraic topology chez Élie Cartan (1869–1951) (*le pere d'Henri*). According to the rumor, the manuscript was abandoned when the doctoral theses of Jean-Pierre Serre (1926–) and Armand Borel (1923–2003) were published. Serre's and Borel's subsequent papers did change the focus of research in topology, away from differential geometric methods to more algebraic methods, principally the spectral sequence and the Steenrod algebra, making the manuscript obsolete. So what was in this manuscript? Could

I get a look at it? The historian salivates at the chance to look at the state of affairs before and after a key event.

Well, the manuscript wasn't there, if, in fact, it exists at all. The archival work I was able to do, however, offered many insights into the workings and spirit of Bourbaki and I will relate some findings in this report. As my story unfurls, I want to consider the allure of the axiomatic method before and after Bourbaki, one of the features of their exposition that has received criticism.

Who is Bourbaki?

> *His name is Greek, his nationality is French and his history is curious. He is one of the most influential mathematicians of the 20th century. The legends about him are many, and they are growing every day. ... The strangest fact about him, however, is that he doesn't exist.*
>
> Paul Halmos, 1957

André Weil was on the faculty at the University of Strasbourg in 1934, together with Henri Cartan. They were responsible for the course on the differential and integral calculus, one of three standard courses required for the *license de mathématiques*, along with general physics and rational mechanics. The standard text was *Cours d'Analyse mathématique* by ÉDUOARD GOURSAT (1858–1936), written before the First World War. Cartan found it wanting, incomplete where generalizations were known, and simply not the best way to present these topics. An explicit example, one with a story of its own, is the formulation of Stokes's Theorem. It may be written

$$\int_{\partial X} \omega = \int_X d\omega,$$

where ω is a differential form, $d\omega$ its exterior derivative, X the domain of integration and ∂X the boundary of X. When everything in sight is smooth, the proof is clear, but the importance of this formula in the case of more general domains of integration is the content of the celebrated theorem of GEORGES DE RHAM (1903–1990), proved in 1931 to answer a question of Elie Cartan relating invariant integrals on Lie groups to the topology of such manifolds.

Persistent badgering by Cartan led Weil to suggest that they write a textbook that they could be satisfied with. Weil writes that he told Cartan, "Why don't we get together and settle such matters once and for all, and you won't plague me with your questions any more?"

The first meeting on 10 December 1934 in Paris to plan the book occurred after a meeting of *le Seminaire Julia*, another of Weil's and Cartan's efforts to fill the gap left in French mathematics after World War I, which Weil called "hectatomb of 1914–1918 which had slaughtered virtually an entire generation" of French mathematicians. The seminar, organized by these young turcs in imitation of the seminars in Germany, needed a sponsor in order to get a room at the Sorbonne. GASTON JULIA (1893–1978) had been the youngest of their teachers at the *École Normale Supérieure* and he stepped up to sponsor them. The seminar treated a topic a year, beginning in 1933-34 with groups and algebras, going on to

Hilbert spaces, then topology. The seminar continued until 1939 when it was superseded by the Seminar Bourbaki.

The committee's first plans were for a text in analysis, that would, according to Weil, "*fix the curriculum for 25 years for differential and integral calculus.*" This text should be *aussi moderne que possible*, *un traité utile à tous*, and finally, *aussi robustes et aussi universels que possible.* Weil already knew a potential publisher in his friend Enriques Freymann, a Mexican diplomat who married the daughter of the founder of *Maisson Hermann*, a scientific publisher. Freymann became the chief editor and manager of the publishing house.

Among the innovations of this text was the suggestion, insisted on by Delsarte, that it be written collectively without *expert leadership.* The initial expectation was that the text would run to 1000–1200 pages and be done in about six months. The initial group of six was expanded to nine members in January 1935, with PAUL DUBREIL (1904–1994), JEAN LERAY (1906–1998) and SZOLEM MANDELBROJT (1899-1983) added. Dubreil and Leray were replaced by JEAN COULOMB and CHARLES EHRESMANN (1905–1979) before the first summer workshop in July, 1935.

The first Bourbaki congress was held in Besse-en-Chandesse in the Vosges mountains. At this workshop, the proposal was made to expand the project to add a *paquet abstrait*, treating abstract (new and modern) notions that would support analysis. These included abstract set theory, algebra, especially differential forms, and topology, with particular emphasis on existence theorems (Leray).

The *paquet* eventually became the *Fascicule de Résultats*, a summary of useful results presented in such a way that a competent mathematician could see where a desired result might be found, and provide the result themselves if they needed it. In fact, the last publication, *Fascicule XXXVI*, part two of *Variétés différentielles et analytiques*, is such a summary. By the way, it is in *Fascicule XXXVI* that the statement of Stokes's Theorem found its place.

During the first conference, with a group of young, eager, and able mathematicians in one place, a new result on measures on a topological space was proved. A note was written up to submit to *Comptes-Rendues.* The name of *Bourbaki* for the group was based on a story out of school: In 1923, Delsarte, Cartan, and Weil were members of the newly matriculated class at *École Normale Superieure*, when they received a lecture notice by a professor with a vaguely Scandinavian name, for which attendance was strongly recommended. The speaker was a prankster, RAOUL HUSSON, wearing a false beard and speaking with an undefinable accent. Taking off from classical function theory, the talk had its climax in *Bourbaki's Theorem* leaving the audience "speechless with amazement." (This Bourbaki was the general who traveled with Napoleon.) Weil recalled this story and the family name was adopted. But why Nicolas? For the submission of the paper, the author needed a *prenom.* It was Weil's wife Eveline who christened the new Bourbaki Nicolas. The note was handled at the *Académie des Sciences* by Élie Cartan who stood up for the unfortunate Poldevian mathematician. The note was accepted and published.

To produce the constituent parts of *les Éléments*, the method of editing adopted by the Bourbaki emphasized communal involvement. A text was brought before a meeting and presented, page by page, line by line, to the group who then expressed any and all criticism. A revision was handed over to another member of the group and the process repeated when a new draft was available. After enough iterations to obtain unanimous approval—either for the strength of the text or the fatigue of the group with the topic—the text would be finalized (usually by Dieudonné) and sent to the publisher.

Digression: The Axiomatic Method

> *In spite of the high pedagogic value of the genetic method, the axiomatic method has the advantage of providing a conclusive exposition and full logical confidence to the contents of our knowledge.*
>
> David Hilbert, 1900

During his 'apprenticeship' (documented in [Weil]), Weil traveled extensively, spending time in Germany while the rise of National Socialism to power took place. As he was interested in number theory, he admired the mathematics of the German schools, especially the axiomatic approach led by the work of David HIlbert (1862–1943) and the Göttingen school. French mathematics through the nineteenth century and into the twentieth was dominated by analysis. Even results of a number-theoretic nature were proved through analytic means. The success of Hilbert's ideas in many fields attracted mathematicians everywhere and so, when looking for a model to shape their project, the members of Bourbaki turned to the axiomatic method.

This phenomenon was not without precedent. When E.H. Moore (1862–1932) came to lead the University of Chicago mathematics department around 1900, he consciously adopted the style of Hilbert's *Grundlagen der Geometrie* as modern, precise, and a model to be imitated. His earliest students at Chicago included Oswald Veblen (1880–1960), Frederick Owens, and R.L. Moore (1882–1974) whose PhD theses concerned the foundations of geometry, axiom systems, and the economy of expression Hilbert achieved. The goals of some of this work were to tighten the systems of axioms describing geometry, to root out redundancy and present the least one needs to assume to achieve Euclid's bounty. These goals, however, though laudable, do not exhaust the depth of the axiomatic method.

Roughly speaking, the *axiomatic method* is an approach to producing mathematics that presents, after some analysis, a set of axioms from which a set of theorems is deduced. The goal in presenting the *right set of axioms* is to avoid deception by intuition. Hilbert's experience with algebraic number theory (his *Zahlbericht*) and invariant theory led him to tread a path leading to more abstract generalization.

When he turned to elementary geometry in his lectures of 1898–99, students in Göttingen were surprised. Already in his early career, Hilbert had remarked of geometry, "One must be able to say at all times—instead of points, straight lines, and planes—tables, chairs, and beer mugs." His stated goal in the *Grundlagen* was " to attempt to choose for geometry a *simple* and *complete* set of *independent* axioms and to deduce from these the most important geometrical theorems in such a manner as to bring out as clearly as possible the significance of the different groups of axioms and the scope of the conclusions to be derived from the individual axioms."

The *Grundlagen* was an immediate success, drawing the following reaction from Henri Poincaré (1858–1912): "The logical point of view alone appears to interest Professor

Hilbert. Being given a sequence of propositions, he finds that all follow logically from the first. With the foundation of this first proposition, with its psychological origin, he does not concern himself .... The axioms are postulated; we do not know from whence they come; it is then as easy to postulate A as C .... His work is thus incomplete, but this is not a criticism I make against him. Incomplete one must indeed resign oneself to be. It is enough that he has made the philosophy of mathematics take a step forward ...."

The philosophical and foundational aspects of Hilbert's efforts are clear. However, the mathematical aspects are not the focus of most discussions of the *Grundlagen*. Among the exercises in independence he has introduced new objects—in particular, non-Archimedean geometries. By isolating the relations among axiom groups, one can discover how the failure of one or more of the assumptions produces new results, the model of this activity being non-Euclidean geometry. His experience in algebra and number theory also supported this view, that the axiomatic method sharpened one's tools with which to craft new arguments, discover new phenomena, and retain the past in a tidy manner to boot.

Another Göttingen product of importance to Bourbaki is in the same spirit: *Moderne Algebra* by B.L. VAN DER WAERDEN (1903–1996) first appeared in 1930, giving an organized account of algebra based on axioms that revealed the similarity in approaches to certain results. The notion of isomorphism plays an important role in algebra and later surfaces as a leitmotif for Bourbaki.

It is important to see that Hilbert and van der Waerden, though formal in presentation, really sought mathematical goals that were not about the past, to recover a complete description of a known theory, but were forward-looking, providing the mathematician with a slim but firm scaffolding on which many new results could be built. The degree to which this view became part of the manner in which modern mathematics was done can be measured by the natural feel we have today for this sort of presentation. It was not always so.

Algebraic Topology chez Bourbaki

> *A côté des structures algébriques (groupes, anneaux, corps, etc.) ... dans toutes les parties de l'Analyse, des structures d'une autre sorte : ce sont celles où l'on donne un sens mathématique aux notions intuitives de limite, de continuité et de voisinage.*
>
> Bourbaki, Topologie 1965

The goal of producing a modern, robust, and universal text led to the most characteristic quality of Bourbaki—a topic was discussed repeatedly in an effort to "digest mathematics, to go to the essential points, and reformulate the math in a more comprehensive and conceptual way [Borel]." The sessions were animated to achieve this goal; after the war, there is a record in *La Tribu* of the rebirth of what were considered classic duels between Cartan and Dieudonné. With their work style and clear goal, "whatever was accepted would be incorporated without any credit to the author. Altogether, a truly unselfish, anonymous, demanding work by people striving to give the best possible exposition of basic mathematics, moved by their belief in its unity and ultimate simplicity [Borel]."

From the first 1935 summer meeting we have the earliest list of topics dates and those responsible for a write-up:

```
Abstract sets (HC)
Algebra (Delsarte)
Real numbers (Dieudonné)
Topology.  Theorems of existence (AW, deP)
Integration
Real functions, series, infinite products
Inequalities:  O and o
Calculus of differential forms
Geometry
Analytic functions:  general part
```

The subject of topology appears in the list and there was a discussion in the spring of 1935 of possible texts that would support their presentation. The classic books by Kerekjarto, Seifert and Threlfall, and Kuratowski were mentioned (none in French). In the first issues of the *Journal de Bourbaki* (later to become *La Tribu*), edited by Delsarte, it was reported that Weil was reading the newly published *Topologie I* of Alexandroff and Hopf, and this text was expected to help them avoid any errors in their presentations. The team writing the topology section, Weil, de Possel, and Henri Cartan are reported in 1936 to be reading (Weil), sleeping (de Possel), or to have written nothing but still thinking about it (Cartan).

The earliest references to 'algebraic topology' in the reports to Bourbaki use the term to refer to duality in topological groups—a discussion later to become 'topological algebra.' In the 1930's the essential points of combinatorial topology was discussed among the Bourbaki: already at the summer conference of 1935, an outline by Weil includes dimension, intersection, linking, degree of mappings, and the index of fixed points among the combinatorial topics. The fundamental group (*groupe de Poincaré*) and covering surfaces were also included. By 1938, Weil made a report on degree and combinatorial topology.

By 1937 there was a plan for the first volumes together with a target date—completion of the first volume by 1.I.1938. The *paquet abstrait* had grown to include the topics of set theory, algebra, set-theoretic topology and abstract integration. In fact, in keeping with the goal of producing a toolbox for mathematicians, the first publication was not a textbook but a list of results (*un fascicule de résultats sans demonstations*) on set theory. Beginning the march toward analysis, it was agreed that set theory served as a basis for future volumes.

Plans for the future volumes were discussed in the *Journal de Bourbaki* until 1940 when the *Journal* was replaced by *La Tribu (Bullétin, apériodique et bourbachique)*. By the time of *La Tribu* the use of the notion of *structure* dominated the formulation of the publishing project. As described later in Bourbaki's entry in Le Lionnais's *Les grands courants de la pensée mathématique*, there were 'mother-structures,' simplest and shared by many mathematical activities; beyond this, one finds 'multiple structures' which blend some number of the mother-structures, for example, topological groups blend the group structure with continuity, while order structures together with algebraic structures give rise to the study of ideals and to integration.

It is this organization by structures that is Bourbaki's lasting legacy. The influence of this notion was far-reaching, even including a psychological discussion of development by

Jean Piaget that cites a correspondence between the mother-structures and a child's first forms of interaction with the world.

Based on the hierarchy of structures, the plan for the *Élements de Mathématique* broke into parts. Part I dealt with the fundamental structures of analysis. In *La Tribu* of 3–15.IX.1940, Part II treated linear analysis, Part III algebraic analysis (to include elliptic functions, the theory of numbers), and Part IV differential topology. We find algebraic topology (that is, combinatorial topology) in this scheme in Part I.

```
Book 1.  Set theory
Book 2.  Algebra
Book 3.  General topology
Book 4.  Topological vector spaces
Book 5.  Elementary techniques of infinitesimal calculus
Book 6.  Integration
Book 7.  Combinatorial topology
Book 8.  Differentials
Book 9.  Calculus of variation
Book 10.  Analytic functions
```

A 25 page report on the shape of books 3 and 7 was titled *Topologia Bourbachica* in which the main topics were I. general topology, 2. topological degree, 3. covering spaces and the Poincaré group, and 4. combinatorial topology (surfaces, Betti groups, Euler-Poincaré formula, indices of vector fields).

Weil was reported to be 'meditating' on the subject of Books 7 and 8, while Ehresmann was working on parts 3 and 4 of Book 7. In late 1941, these books were listed as urgently in need of work, *"la rédaction a le regret ... que ces livres brillent toujours par leur inexistence."* *

The summer meeting of 1942 (in Clermont) presented a new organization of Part I:

```
1.  Sets
2.  Algebra
3.  General Topology
4.  Functions of a real variable (elementary theory)
5.  Combinatorial topology
6.  Topological vector spaces
7.  Differential calculus and manifolds
8.  Integral calculus and differential forms
9.  Analytic functions
```

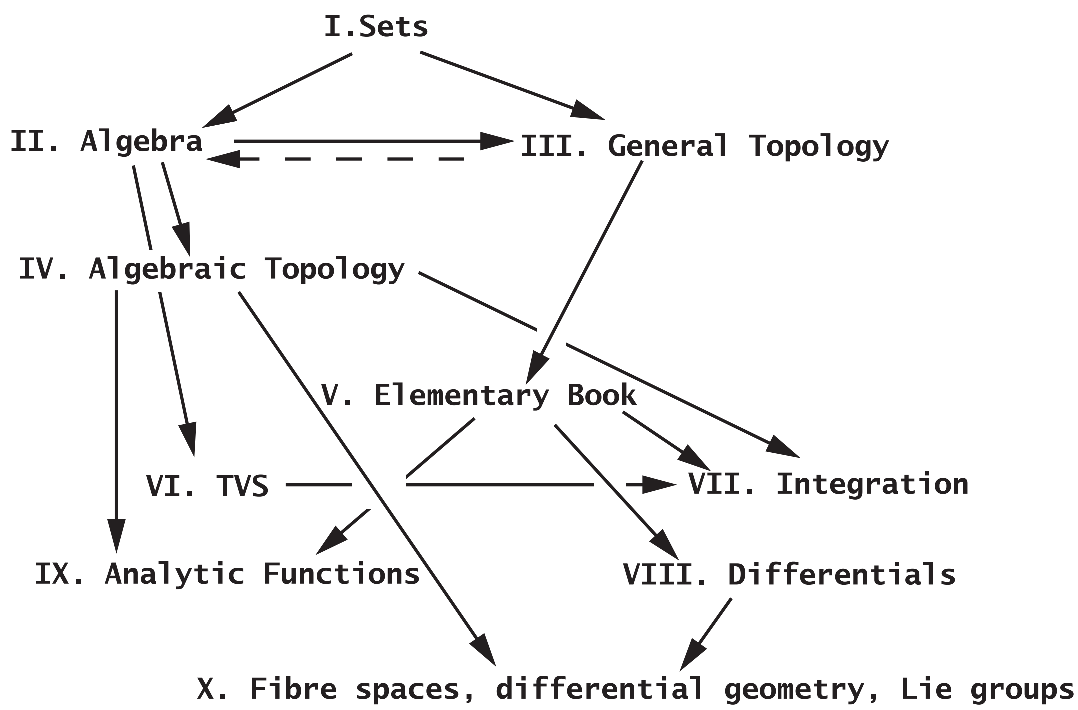
On this plan little progress on algebraic topology took place. In *La Tribu* no. 10 of 10–15.IV.1944, it is reported that *"le récent Congrés Bourbaki que s'est tenu à Paris du 6*

* "the editors regret ... that these books are conspicuous by their nonexistence."

au 8 Avril 1944 n'on a pas moins réalisé au progrés important et depuis longtemps souhaité par la rédaction: le demarrage de la Topologie algébrique."[*]

A description of the core of the subject at the time was given, however: a) there should be no Menger theory of curves, no graphs, no Peano continua, no continua; b) a chapter on knots; c) higher homotopy groups and fibre spaces, which they deemed interesting, having a future, but at present in a state *"trop larvaire."* The development of this topic took place during the war with the work of Ehresmann, Feldbau, Cartan, and Leray in France, Steenrod and Whitney in the US, and Hopf and Eckmann in Switzerland (see [McCleary]).

La Tribu of 11–15 July 1945 contains a picture of the dependencies among topics in Part I, once again featuring algebraic topology near the foundations.



11-15.IV.1945 Congress in Paris, from La Tribu no. 8

The 1947 organization of the general plan changed again—the basics now broke up into blocs:

```
                         General Plan
I. Sets, II. Algebra, III. General Topology
Linear bloc:  IV. Functions of a real variable,
    V. Topological vector spaces, VI. Integration, VII. Local differentials
Topologico-differential bloc:  VIII. Algebraic topology, IX. Manifolds,
    X. Lie groups
```

[*] "the recent Bourbaki Congress that was held in Paris from the 6th to the 8th of April 1944 nevertheless realized important progress, long wished for by the editors: the beginning of algebraic topology."

In 1946, with the end of World War II, and travel easier, SAMUEL EILENBERG (1913–1998) was drafted as a member, explicitly to prepare a report on algebraic topology. By 1949 there was an 82-page document, `Rapport SEAW sur la topologie préhomologique`, by Eilenberg and Weil, treating the important aspects of the topology of fibre spaces. This densely written report developed the point-set properties of fibre spaces, including some new ideas. For example, they defined the *épiderme* of a space (with the parenthetical remark, *pourquoi pas*); this "skin" is a covering of the space with good properties of extension.

It is the 1950 Grand Plan that gives the familiar list of topics to be treated:

```
Part I.
1.  Sets
2.  Algebra
3.  General topology
3bis.  Geometric topology
4.  Functions of a real variable
5.  Topological vector spaces
6.  Integration
7.  Manifolds
8.  Analytic functions
9.  Lie groups
```

Part II treated Commutative Algebra, Part III Algebraic Topology and its applications, and Part IV Functional Analysis.

The new topic, Geometric topology, was named by Serre to treat topics like coverings, fibre spaces, homotopy, polyhedra, retracts, and the fundamental group. This term went on in the literature, but it did not sit well with the Bourbaki who coined other terms to mock it.

So What Happened?

> *Furthermore, in a time in which indiscriminate use of science and technology threatens the future of the human race, or at least the future of what we now call civilization, it is surely essential that a well integrated report about our mathematical endeavors be written and kept for the use of a later day "Renaissance."*
>
> Pierre Samuel, 1972

Another French enterprise was born about this time that affected the efforts to bring a text on algebraic topology together. In 1948/49, the *Séminaire Henri Cartan* began in Paris. Cartan had just come from Harvard in 1948, having spoken on topological notions, especially what later became sheaves. From its inception the seminar treated topological themes, beginning with basic notions in 48/49 and going on to treat fibre spaces, spectral sequences, sheaves, homology of groups and Eilenberg-Mac Lane spaces, in later years. The level of exposition of these lectures was consistent with the expectations of the Bourbaki, and many of the lectures were given by then current members of Bourbaki.

The discussions of algebraic topology in the earliest plans for *Élements de mathématique* and its appearance among the basic tools for the intended audience of Bourbaki make it clear what status the topic had for the group. However, the development of the subject was so rapid in the post-war years that it could not be understood in the manner that the Bourbaki set as a standard for their published work—that the essential concepts be identified, and the axiomatic basis presented in such a way that the main theorems would be smoothly proven from first principles. The collateral development of homological algebra, which would provide a tool for algebraic topology was finally taken up by Bourbaki, but only in recent times (1980). It is significant that some of this development was carried out by members of Bourbaki itself—Cartan, Eilenberg, Serre, Borel, and others. The press of new discoveries caused Bourbaki to wait.

The published work of Bourbaki does not make for easy reading. The austere style is associated with a monolithic view of the unity of mathematics that is precisely and properly presented in their work. The philosophical cadre of "structure" as guidepost and goal makes for a good explanation of the finished product. However, the record of the archives tells a different story. The austerity is a result of group editing. The course of a document was almost chaotic from first presentation to final publication, spiced by the lively interchanges of mathematicians of the first order, committed to an extraordinary standard.

From the point of view of an enterprise, Bourbaki's *Élements* stands out as an effort to rebuild a mathematical culture, based on a method (the axiomatic method) that was seen to be fruitful, by a collective of gifted mathematicians whose anonymity in their work was offset by the *joie de vivre* the process involved. We should all be so moved to do the same. And I wonder what kind of report on algebraic topology we would produce today.

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Algebraic topology was axiomatized by Samuel Eilenberg , a Polish-born American mathematician and Bourbaki member, and the American mathematician Norman Steenrod. Saunders Mac Lane , also of the United States, and Eilenberg extended this axiomatic approach until many types of mathematical structures were presented in families, called categories. Hence there was a category consisting of all groups and all maps between them that preserve multiplication, and there was another category of all topological spaces and all continuous maps between them. To do algebraic topology was to transfer a proble Algebraic topology is a branch of mathematics which uses tools from abstract algebra to study topological spaces. The basic goal is to find algebraic invariants that classify topological spaces up to homeomorphism. In many situations this is too much to hope for and it is more prudent to aim for a more modest goal, classification up to homotopy equivalence. Algebraic topology is the mathematical machinery that lets us quantify and detect this. The idea behind algebraic topology is to map topological spaces into groups (or other algebraic structures) in such a way that continuous functions between topological spaces map to homomorphisms between their associated groups.1.Â Each has a dierent method for dening a group from the structures in a topological space, and although there are close links between the three, they capture dierent qual-ities of a space. Many of the more advanced topics in algebraic topology involve studying functions on a space, so we introduce the fundamental link between critical points of a function and the topology of its domain in Section 5 on Morse Theory.