Algebra in the House of Wisdom

Johar M. Ashfaque

Abstract

We study the history of algebra, a selected topic from the history of mathematics, during the era of the House of Wisdom. We discuss the contributions made by al-Khwarizmi, Alhazen and Omar Khayyam. al-Khwarizmi being of particular interest since he wrote the treatise on algebra. Alhazen and Omar Khayyam succeeded al-Khwarizmi, and extended al-Khwarizmi's work on algebra: the former worked on conic sections whilst the later worked on finding the solutions of cubic equations.

1 Introduction

The late 8th century saw the ascension of the rule of the Abbasid caliphate in Baghdad. The House of Wisdom was a library as well as a translation institute, established in Baghdad during the reign of the caliph al-Mamun. At the orders of al-Mamun, many scholars were invited to visit the House of Wisdom in its day to translate the manuscripts sent to him by the Byzantine emperor. One of the scholars was al-Khwarizmi, employed by al-Mamun, who worked as a mathematician and as an astronomer, best known for his treatise on algebra.

However, the fierce Mongol invasion of Baghdad in the late 13th century, bore witness to the House of Wisdom along with all the other libraries being destroyed, which led to the end of the Abbasid rule instantly. For further details, see [1, 2].

The aim of this paper is to discuss the treatise of al-Khwarizmi, Alhazen and Omar Khayyam, in order to realise the sheer importance of their work in mathematics.

2 Treatise of al-Khwarizmi

Abu Ja'far Muhammad ibn Musa al-Khwarizmi, better known as al-Khwarizmi worked as a mathematician and as an astronomer, being employed by the caliph al-Mamun, in the House of Wisdom. As a mathematician, al-Khwarizmi was instrumental in introducing the Arabs to the Hindu numerals [3]. But his finest and greatest work was his treatise on algebra: *Hisab al-jabr w'al-muqabala*, translated as *The Compendious Book on Calculation by Completion and Balancing*. The word *algebra* is derived from the name of one of the basic operations with equations, al-jabr, meaning adding equal terms to both sides of an equation in order to eliminate the negative terms, described in this treatise. The word *algorithm* stems from Algoritmi, the Latin form of his name, see [3].

The numerals in his treatise are expressed as words. al-Khwarizmi through the use of examples alone, explains his methods of finding the solutions therefore giving no general solution. al-Khwarizmi explains the solution of six types, [5], reducing all linear and quadratic equations to one of these forms:

$$ax^2 = bx \tag{1}$$
$$ax^2 = b \tag{2}$$
$$ax = b \tag{3}$$
$$ax^2 + bx = c \tag{4}$$
$$ax^2 + c = bx \tag{5}$$
$$ax^2 = bx + c \tag{6}$$

where a, b, and c are given positive numbers. For example, suppose we are given a quadratic equation

$$5x^2 = 20x - 15x^2.$$

Using the operation, al-jabr, we add $15x^2$ to both sides to obtain

$$20x^2 = 20x,$$

which reduces to

$$x^2 = x. \tag{7}$$

We can see that 7 satisfies one of the forms deduced by al-Khwarizmi. A section in his treatise also contains problems with solutions involving only simple arithmetic or linear equations, based on the Islamic law of inheritance, see [3, 5].

Most of al-Khwarizmi's work has either been lost or destroyed, as a result what we know about al-Khwarizmi's work is based upon secondary resources. For detailed references, see [5].

3 Alhazen, Optics and Conics

Abu Ali al-Hasan Ibn al-Haytham, known in the West as Alhazen [3]. Alhazen is remembered for his treatise *Kitab al-Manāthir* (Book of Optics) [3], which provided the first partial solution to the problem arising in geometry due to the reflection of light from curved mirrors. Alhazen, solved this problem by finding the solutions to the quartic equation, by defining equations for the intersection of the two conic sections: a circle and a hyperbola [3]. The points of intersection formed a solution set, satisfying the quartic equation, therefore solving the problem. Alhazen also knew about calculus and gravity, explains [1], computing the interval of the twilight. Refer to [3, 6], for further details.

4 Omar Khayyam and the Cubic Equations

Omar Khayyam, unlike other scholars who wrote in Arabic, wrote in Persian. He was thereby considered to be a Sufi mystic [3]. Khayyam's greatest contribution in mathematics was his famous *Treastise on Demonstration of Problems of Algebra* [3], which classifies the different types of cubic equations and provides a general theory for their solution.

Omar Khayyam in his treatise, explains [5], considered cubic equations of the form:

$$x^3 + ax = b \tag{8}$$
$$x^3 + b = ax \tag{9}$$
$$x^3 = ax + b \tag{10}$$
$$x^3 + ax^2 = b \tag{11}$$
$$x^3 + b = ax^2 \tag{12}$$
$$x^3 = ax^2 + bx \tag{13}$$

where a, b and c are given positive numbers. He developed further methods, both algebraic and geometric for solving the cubic equations systematically, based on conic sections [5]. Omar Khayyam also gave important results on ratios in his treatise [5], extending Euclid's work on the multiplication of ratios.

In his book of algebra, Omar Khayyam refers to one of his other works which is lost [3]. His lost work was based on the Pascal triangle but he was not the first, since al-Karaji discussed the Pascal triangle before this date [3]. Omar Khayyam also posed a question as [5] explains, of whether ratios can be considered to be a number, leaving the question unanswered.

5 The Great Fall of the House of Wisdom

During the late 13th century, the invasion of Baghdad by the Mongols saw the House of Wisdom along with all the other libraries being destroyed, [1]. The magnificent collection of books and manuscripts stored in the House of Wisdom were thrown into the River Tigris, explains [1], which ran black for months as a result of the ink from the books and the manuscripts.

6 Conclusion

In this paper we have briefly discussed an era, which many seem to have forgotten, during which the House of Wisdom was established. We have briefly discussed the treatise of al-Khwarizmi, Alhazen and Omar Khayyam and the impact of their work in mathematics. For further reading, I highly recommend [1, 3, 5, 6].

A Conic Sections

Type	General Equation
Hyperbola	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
Ellipse	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
Circle	$(x-a)^2 + (y-b)^2 = r^2$
Parabola	$y^2 = 4ax$

For further study on conic sections, refer to [7].

References

[1] M.H. Morgan, *Lost History: The Enduring Legacy of Muslim Scientists, Thinkers, and Artists,* National Geographic Society, July 2007.

[2] V. Sanford, *A Short History Of Mathematics,* G.G. Harrap, January 1930.

[3] J. Al-Khalili, *Pathfinders: The Golden Age Of Arabic Science,* Allen Lane, September 2010.

[4] D. Knuth *Algorithms in Modern Mathematics and Computer Science,* Springer-Verlag, November 1981.

[5] B.L. Waerden, *A History Of Algebra,* Springer-Verlag, August 1985.

[6] I. Al-Haytham and J.P. Hogendijk, *Ibn Al-Haytham's Completion of the Conics,* Springer-Verlag New York Inc., January 1985.

[7] J. Roe, *Elementary Geometry,* Clarendon Press, March 1993.

It was so notable that it eventually found its way into Europe, becoming the first text book on the subject of Algebra in Europe. The Europeans eventually used the name â€œal-jabrâ€ for the name of this subject (which in the translated Latin text version was â€œalgebraeâ€, hence â€œalgebraâ€).Â One of the first Directors of the House of Wisdom in Bagdad in the early 9th Century was an outstanding Persian mathematician called Muhammad Al-Khwarizmi. He oversaw the translation of the major Greek and Indian mathematical and astronomy works (including those of Brahmagupta) into Arabic, and produced original work which had a lasting influence on the advance of Muslim and (after his works spread to Europe through Latin translations in the 12th Century) later European mathematics. algebra was due to his â€œnot want[ing]. to offend the caliph whose patronage. he enjoyedâ€ (111), completely overÂ This article reviews The House of Wisdom: How Arabic Science Saved Ancient Knowledge and Gave Us the Renaissance. by Jim Al-Khalili 302 pp. , New York, 2011. Price: $29.95 (paper) ISBN 978-1-59420-279-7. Read more. "For want of the necessary notation the Greeks had no algebra in our sense. They were obliged to use geometry as a substitute for algebraical operations; and the result is that a large part of their geometry may appropriately be called 'geometrical algebra'." So for Heath, referring to the diagram above, and in modern notation,$$ \mbox{ if } AX = a, ~XB = b ~\mbox{and} ~PX = x,~ \mbox{then}~ ab = x^2$$.Â Later, the Caliph al-Ma'mun (813-833) established the 'House of Wisdom', which through the 9th and 10th centuries acted as a centre for the great translation programme of knowledge that scholars had accumulated and had built on through their own discoveries. The House of Wisdom (Arabic: Ø¨ÙŠØª Ø§Ù„ØÙƒÙ…Ø©â€Ž, romanized: Bayt al-á¸¤ikmah), also known as the Grand Library of Baghdad, refers to either a major Abbasid public academy and intellectual center in Baghdad or to a large private library belonging to the Abbasid Caliphs during the Islamic Golden Age. The House of Wisdom is the subject of an active dispute over its functions and existence as a formal academy, an issue complicated by a lack of physical evidence following the collapse of the Abbasid Caliphate and a