# Enlightenment Age Advances in Dynamics and Celestial Mechanics

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## Overview

Using equations based on Newton's laws, 18th century mathematicians were able to develop the symbolism and formulae needed to advance the study of dynamics (the study of motion). An important consequence of these advancements allowed astronomers and mathematicians to more accurately and precisely calculate and describe the real and apparent motions of astronomical bodies (celestial mechanics) as well as to propose the dynamics related to the formation of the solar system. The refined analysis of celestial mechanics carried profound theological and philosophical ramifications in the Age of Enlightenment. Mathematicians and scientists, particularly those associated with French schools of mathematics, argued that if the small perturbations and anomalies in celestial motions could be completely explained by an improved understanding of celestial mechanics, i.e., that the solar system was really stable within defined limits, such a finding mooted the concept of a God required adjust the celestial mechanism.

## Background

Theories surrounding celestial mechanics grew and matured along with the Scientific Revolution and Age of Enlightenment. As 17th and 18th century scientists sought to explain the driving and controlling forces related to celestial motion, the various explanations found favor, including those that treated the planets as gigantic magnets that attracted and repelled each other in a cyclic dance. In his seminal work, *Philosophiae Naturalis Principia Mathematica* (Mathematical Principles of Natural Philosophy) English physicist Sir Isaac Newton's (1642-1727) formulation of the laws of gravitation, however, provided the first comprehensive and mathematically consistent explanation for the behavior of astronomical objects.

In the middle of the 18th century, French mathematician Jean Le Rond d'Alembert (1718-1783) wrote extensively for Denis Diderot's (1713-1784) *Encyclopédie* on various scientific subjects. In addition to the influence of Newton, during his education d'Alembert was heavily exposed to French mathematician René Descartes' (1596-1650) earlier vision of the physical world. According, although he later rejected much of

Descartes' work, the concept of a mechanistic universe that could be described mathematically was an early and formative influence on d'Alembert.

D'Alembert's professional writings were important to clarification of problems in mathematical physics, especially problems related to the Newtonian concepts of kinetic energy. In 1753, d'Alembert published an influential work titled, *Traité de dynamique*, that set forth d'Alembert's principles of mechanics as derived primarily from mathematical analysis instead of observational data. What eventually became known as d'Alembert's principle was a insightful interpretation of Newton's third law of motion and d'Alembert's contributions to the study of dynamics became widely known and influential with his elaboration of Newtonian concepts of force. D'Alembert calculations regarding gravity extended the validity and acceptance of Newton's formulation of the inverse square law of force of gravity.

D'Alembert's philosophy of science tended toward the metaphysical and away from reliance on experimental data. In this regard, D'Alembert clearly ran counter to Enlightenment empiricism. ,,,

French mathematician Joseph-Louis Lagrange (1736-1813), born in Italy under the name Giuseppe Lodovico Lagrangia, also published important works on dynamics based on his principle of least action. Some of the most influential of Lagrange's work appeared between 1759 and 1766 in the journal, *Mélanges de Turin*. Lagrange's work *Mécanique Analytique* (Analytical Mechanics) published in 1788 was notable among scholars for its clarity of notation and it was the first book treating mechanics through purely mathematical analysis without resorting to the aid of diagrams.

In papers on the topic of fluid mechanics Lagrange advanced what would become known as the Lagrangian function and other methodologies to attack problems associated with observations of the orbital dynamics of Jupiter and Saturn….

Another French mathematician, Pierre Simon de Laplace (1749-1827), worked to explain the small discrepancies between Newton's predicted and the observed orbits of the planets. Laplace understood that Newton's calculations had ignored the small yet significant gravitational influences of the other planets in the solar system. Newton largely discounted these perturbations in his mechanistic universe. In fact, Newton and natural theorists explained such aberrations as requiring the hand of God to constantly "wind the celestial watch, lest it run down" or to otherwise "reset" the mechanism of celestial mechanics. Laplace rejected this need for divine intervention, and Laplace strove to fully explain nature along mechanistic and deterministic lines.

In 1771, Laplace's work, *Recherches sur le calcul intégral aux différences infiniment petites, et aux différences finies, c*ontained formulations important to astronomers. In 1773, Laplace published a study titled Traité de mécanique céleste (Traits of Celestial Mechanics) that set out exceedingly detailed mathematical calculations involving the eccentricities of planetary orbits. Significantly, Laplace's work accounted for the gravitational influences caused by multiple celestial bodies (planets) and involved an accounting of their mutual gravitational attraction.

In his 1979 book, *Exposition du système du monde* (The System of the World), Laplace set forth elegant calculations involving the orbital and rotational dynamics of bodies in a gravitational field. Laplace argued an early accretion (addition or gathering) hypothesis that allowed for the creation of the solar system from nebular gas constrained and contracted by gravity into the bodies observable today. Laplace specifically asserted that the planets in the solar system formed from the disruption and debris of a rotating, contracting and cooling solar nebula. In addition, Laplace was able to make very accurate predictions on the future positions of astronomical bodies. Many of Laplace's predictions were later confirmed by astronomers' identification of lunar positions and celestial objects in accord with Laplace's calculations.

In the later portion of the 18th century, Laplace began to develop and incorporate probability theory into his work on celestial mechanics. In 1786 Laplace demonstrated that observed eccentricities and irregularities in planetary orbits remained within predicted and defined limits. More importantly, Laplace showed these systems to be self-correcting.

Other physicists and mathematicians made notable contributions to the understanding of celestial dynamics, Swiss mathematician Leonhard Euler (1707-1783) studied lunar options and made detailed calculations regarding the interactive dynamics of the Sun, Earth & Moon Sun system. Euler also worked on problems associated with perturbations (small changes) in planetary orbits). Most importantly, Euler studied the dynamics of a three-body system in a gravitational field.

## Impact

D'Alembert's metaphysical analysis culminated with his five volume work Mélanges de literature et de philosophie that was published starting in 1753. Although d'Alembert's writing did not deny the existence of a God, his allowance for the existence of God was not based on belief in divine revelation, but rather on d'Alembert's opinion that man's intelligence could not be solely attributed the natural interaction of matter. As he aged, however, d'Alembert gradually became a materialist and discounted Deistic or natural philosophical arguments for a God ruling the mechanistic universe. This shift was to have profound influence on generations of French mathematicians mentored by d'Alembert. As a school, the French mathematicians took and increasingly skeptical or hostile attitude towards arguments in favor of a God needed to intervene in workings of a mechanical universe.

D'Alembert's work, however, established that important physical laws, especially those associated with dynamics and mechanics could in some circumstances still be deduced from pure mathematical analysis. In essence, during an age of empiricism, d'Alembert reasserted a role for purely mathematical analysis. Despite this departure from empiricism, d'Alembet's work helped extend both the range and power of Newtonian physics in 18th century European scientific circles

Laplace's main contribution to the advancement of Newtonian physics was in his translation of Newton's geometrical analysis to a more widely understandable calculus based analysis of mechanical dynamics. Although Laplace became the most important champion of a Newtonian-based understanding of celestial dynamics, French mathematicians argued that Laplace's work essentially removed the need for a god to tinker with -- or reset -- Newton's clockwork universe. Laplace made his assessments of the stability of the solar system by demonstrating the invariability of mean planetary motions (the motions of planets averaged over time).

Laplace's interpretation of celestial mechanics ran counter to philosophical and theological Enlightenment views of celestial mechanics a both proof of God as a "prime mover" and of the continued need for God's existence. Laplace argued for a completely deterministic universe, without a need for the intervention of God. Laplace even asserted explanations for catastrophic events (e.g., flooding, comet impacts, extinctions, etc.) as the inevitable results of time and statistical probability.

The need for greater accuracy and precision in astronomical measurements spurred the development of improved telescopes and pendulum driven clocks. Consequently, the accuracy of mathematical predictions improved with each generation of instruments. More importantly, each generation of new data brought more general confirmation of Newtonian physics.

In general, despite important mathematical advances, observation outpaced prediction during the 18th century (e.g., English astronomer William Herschel's 1781 discovery of Uranus) and mathematicians were left to scramble for explanations consistent the emerging dominance of Newtonian physics. With very minor exceptions these explanations were always found. Accordingly, both observation and calculation accelerated the influence and rise of Newtonian physics as the basis for further advances in dynamics and celestial mechanics. The scientific world would have to await the development of relativity theory in the 20th century to fully explain away the minor discrepancies in Newtonian descriptions of the universe.

Further Reading

Bell, E. T. (1986) Men of Mathematics: The Lives and Achievements of the Great Mathematicians from Zeno to Poincaré. New York: Simon and Schuster.

Boyer, C. B. (1968) A History of Mathematics, 2nd ed. New York: Wiley.

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Enlightenment philosophy and Great Awakening Christianity were very different, but both influenced the American colonies and American Revolution and both frame our thinking today. The Enlightenment â€” so named by its own practitioners, who didnâ€™t lack self-esteem â€” is best thought of as a continuation of the Renaissance we read about in Chapter 2, with a strong emphasis on the Scientific Revolution, reason, and progress. Its practitioners adhered to the scientific method of testing hypotheses through rigorous, repeatable experimentation. Ancient Greeks, inventors of the first organized sporting The Age of Enlightenment, also known as the Enlightenment, was a philosophical movement that dominated the world of ideas in Europe in the 18th century. Centered on the idea that reason is the primary source of authority and legitimacy, this movement advocated such ideals as liberty, progress, tolerance, fraternity, constitutional government, and separation of church and state.Â  The cultural exchange during the Age of Enlightenment ran in both directions across the Atlantic. In their development of the ideas of natural freedom, Europeans and American thinkers drew from American Indian cultural practices and beliefs. First page of the Encyclopedie published between 1751 and 1766. Enlightenment thinkers in Britain, in France and throughout Europe questioned traditional authority and embraced the notion that humanity could be improved through rational change. The Enlightenment produced numerous books, essays, inventions, scientific discoveries, laws, wars and revolutions. The American and French Revolutions were directly inspired by Enlightenment ideals and respectively marked the peak of its influence and the beginning of its decline. The Enlightenment ultimately gave way to 19th-century Romanticism. The Early Enlightenment: 1685-1730. Starting from 18 November 2020, all authors who publish in CELESTIAL MECHANICS AND DYNAMICAL ASTRONOMY will now sign a License to Publish (LTP) form rather than a copyright transfer form. LTP allows authors to retain both copyright and the intellectual rights to their work. Authors also retain ownership and the moral rights to their research, and do not have to request permission for specific uses of their own materials, so long as it is credited to the original source. We look forward to receive your submissions to CELESTIAL MECHANICS AND DYNAMICAL ASTRONOMY: https://www.springer.com/journal/ The international journal Celestial Mechanics and Dynamical Astronomy (CM&DA) is concerned with the broad topic of celestial mechanics and its applications, as well as with peripheral disciplines. It is the journal of record in its field and is an indispensable component of reference libraries on Dynamical Astronomy, Astrodynamics and Dynamical Systems. Celebrating its 50th anniversary in 2019 with a dedicated article collection. Offers wide-ranging coverage of celestial mechanics and related fields. Covers the mathematical, physical and computational aspects of planetary theory, lunar theory