From Berkeley to Bourbaki *

Alasdair Urquhart
University of Toronto

This has been a great century for logic and the foundations of mathematics. Ewald's excellent source book is a very welcome addition to the literature on the exciting developments of this and the last two centuries. The richness of the material on which Ewald is drawing is shown by the fact that he has assembled a broad and representative selection without once duplicating anything to be found in the famous source books of van Heijenoort and Benacerraf/Putnam.

Ewald's aims are different from those of the earlier source books, which were devoted to the modern period in foundational research, from the birth of modern logic in Frege to the demise of the classical Hilbert programme at the hands of Gödel. Ewald sees the modern period in foundations as falling into two periods, the first beginning with Kant and lasting until Hilbert, the second beginning with Frege and continuing to the present day. His source book is devoted to the first of these periods. Its chronological scope, however, is broader than this classification suggests, since it begins with Berkeley's notebooks of 1707-8, and ends with a manifesto of Nicholas Bourbaki from 1948.

The editor has aimed to include documents that are either difficult to obtain or have been unaccountably neglected; many important pieces are translated here for the first time. He avoided duplicating work in the earlier source books, which explains otherwise puzzling omissions; for example, there is only one very short piece by Frege, and Russell and Gödel are entirely missing. To obtain a full picture of the period covered by the volumes, it would be necessary to supplement them with the earlier collections of readings. Nevertheless, they form a tremendously rich resource.

The first volume makes easily available for the first time the *Analyst*, Berkeley's remarkable polemical attack on an "infidel mathematician" (probably Edmund Halley). With deadly acumen, Berkeley puts his finger on the weakest spots in the foundations of Newtonian infinitesimal analysis. A fully satisfactory reply to Berkeley's strictures was not found until the development of the doctrine of limits in the nineteenth century (some might even say that the matter was not really elucidated until the twentieth century, with the invention of

*William B. Ewald, *From Kant to Hilbert: A Source Book in the Foundations of Mathematics* (Oxford: Clarendon Press, 1996). Two volume set. xviii + 1340 pp. $526.50, set. Page references are to this work.

non-standard analysis). However, Berkeley's attacks did not go unanswered at the time. Ewald reproduces selections from the most elaborate contemporary defence of Newtonian calculus, Colin MacLaurin's *Treatise on Fluxions*. This attempt at shoring up the foundations consists essentially in taking the concepts of physical motion and velocity as primitive, something quite foreign to the classical Greek way of looking at things, though the book bears the subtitle: "Demonstrated after the Manner of Ancient Geometricians."

With Bernard Bolzano, we enter an entirely new era in the foundations of analysis, and it is appropriate that the editor provides generous selections from this great writer. The essay *Contributions to a better-grounded presentation of mathematics*, written in 1810, but anticipating doctrines (such as anti-psychologism) later associated with Frege, is translated here for the first time. Also included is the famous 1817 "purely analytic" proof of the intermediate value theorem. The rigour and clarity of Bolzano's proof is astonishing, given its early date.

The problems of analysis were not of course the only foundational problems of the nineteenth century. Lively discussions took place about the nature of geometry and method in algebra.

The selections on geometry in Ewald's first volume are limited to Lambert, Gauss and Clifford; this seems a little unfair to Bolyai and Lobachevsky. The latter at least had the courage of their convictions in stating publicly the possibility of a geometry contradicting the Euclidean axiom of parallels, whereas Gauss confined himself to private letters, justifying his intellectual cowardice by saying that he wanted to avoid the "clamour of the Boeotians." He told Bolyai that he couldn't praise his work, as he would be praising himself – a dog-in-the-manger attitude that Bolyai never forgave.

In the middle of the nineteenth century the British algebraists Peacock, De Morgan, Gregory, Hamilton and others groped their way slowly towards a purely formal conception of algebraic inference. The editor provides us with a selection from their writings, together with some amusing notes on the surprisingly influential boneheads Francis Maseres and William Frend, who wrote ponderous treatises proving that there are no such things as negative numbers. The mathematical correspondence of De Morgan and Hamilton makes entertaining reading, rambling over mathematical topics from algebra and calculus to geometry.

The most important piece from the British algebraic tradition reprinted here is George Boole's *Mathematical Analysis of Logic*, a seminal treatise that has not been easy of access. Ewald's introductory note expresses a common opinion that Boole's system differs from the later version of Venn by using exclusive rather than inclusive disjunction. But in fact Boole's practice is much odder than this remark suggests. Boole not only operates with his algebraic expressions as if the whole apparatus of classical algebra is applicable, but also uses division and other operations that do not make sense in what we now know as "Boolean algebras." Theodore Hailperin in his book *Boole's logic and probability* has made a valiant attempt at sorting out this mathematical tangle.

The omission of Frege is amply compensated by a fine selection from Charles Sanders Peirce, whose neglect the editor rightly bemoans. It is very good to have easily available his pioneering essay of 1885, *On the algebra of logic*, setting out the basics of quantification theory with great clarity, and in a notation that is recognizably the direct ancestor of our current notation. Building on the work of his student O.H. Mitchell, Peirce created the modern predicate calculus (though anticipated by Frege in 1879), and started the modern tradition of algebraic logic that led through Schröder and culminated in the contemporary theory of models.

Ewald's second volume contains a rich selection from the German and French schools. It begins with Riemann's famous and prophetic inaugural lecture on the foundations of geometry (delivered 1854, published 1868), and continues with three of Helmholtz's essays on geometry, perception and measurement. I had not realized until reading the editor's introductory essay to these three pieces how important Helmholtz was in reviving an analytic neo-Kantianism in the foundations of science, and so leading the Germans out of the dismal swamp of Hegelian *Naturphilosophie*. Readers of these essays will not only enjoy Helmholtz's sparkling intellect, but also taste the authentic flavour of Wilhelmine Germany, ruled by an emperor in whom "was united everything that humanity, up to now, has regarded as worthy of veneration and gratitude" (p. 691).

Nine selections from Richard Dedekind follow. The two famous essays on the real and natural numbers, *Continuity and Irrational numbers* and *Was Sind und Was Sollen die Zahlen?* are included, but also less familiar pieces. Notable among the latter are extracts from the famous tenth and eleventh supplements to Dirichlet's *Lectures on the Theory of Numbers*, work that created the field of modern algebra. Dedekind's fundamental idea, whether in ideal theory, or in the foundation of real analysis, was to define new mathematical objects in terms of sets of given simpler objects. Oddly enough, he does not necessarily identify the new objects with the corresponding sets; for example, he maintained that real numbers are not identical with cuts in the rational numbers. In reply to Heinrich Weber, he wrote: "You say that the irrational number is nothing other than the cut itself, while I prefer to create something *new* (different from the cut) that corresponds to the cut and of which I say that it brings forth, creates the cut" (p. 835). Before rushing to condemn Dedekind's idea, as Russell did, we should remember that the alternative view of Weber and Russell (the current conception) also has its difficulties. On the Weber/Russell view, as Dedekind points out, we have to make a wholly artificial distinction between a rational number and the cut it determines.

The Dedekind selections also include a short account by Felix Bernstein of a visit to Dedekind in 1897. Bernstein recorded the following marvellous description of the two friends:

> Dedekind said, with respect to the concept of set, that he imagined a set as a closed sack that contains completely determinate things – but things which one does not see, and of which one knows nothing except that they exist and are determinate. Somewhat later, Cantor gave his own conception of a set. He drew his colossal figure upright, made a magnificent gesture with his raised arm, and said, staring into the indeterminate, 'A set I imagine as an abyss' (p. 836).

In the section devoted to the work of Cantor, his *Grundlagen* and correspondence with Dedekind are translated for the first time. The correspondence (recently rediscovered and now housed in Evansville, Indiana) is particularly fascinating, showing the creation of an entirely new theory by the most imaginative mathematician of his time. The later correspondence in particular, in which Cantor draws the distinction between consistent and inconsistent multiplicities, together with material quoted by the editor, clearly refutes the idea that Cantor operated with a "naïve conception of set," or subscribed to the unlimited comprehension axiom, like Frege. Unlike Russell, Cantor did not see the paradoxes as a threat to set theory, but rather as pointing to distinctions that could lead to progress in the foundations of the subject that he created single-handed.

Cantor's great opponent Leopold Kronecker wrote nothing systematic on the foundations of mathematics, in spite of his strongly held and very influential constructivist convictions. Here he is represented by a quote from Weyl, a polemical attack from Hilbert's Göttingen lectures of 1920 (Kronecker is described as a "radical dictator" who averts his gaze from the progress of mathematics by pursuing an "ostrich-politics"), and a short essay by Kronecker on the concept of number. Kronecker's most famous aphorism, "God created the integers, everything else is the work of man," was not published by Kronecker himself, but seems to have been first reported by Heinrich Weber in 1893.

Felix Klein, like Kronecker, was a critic of the project of "arithmetizing the continuum," but from a completely opposite standpoint. Klein stood for geometrical intuition in the vein of Riemann; he felt it was being forgotten in the trend to Weierstrassian rigour. Ewald reproduces a lecture of 1893 containing a notorious passage in which Klein credits the Teutonic race with "strong naïve space-intuition," while the "critical, purely logical sense" is more fully developed in the Latin and Hebrew races (p. 962). The editor, in a long footnote, convincingly defends Klein from the charges of nationalistic and racial bias that arose from this passage.

A substantial collection of Poincaré's controversial articles on logic and foundations of mathematics are included. These have been easily available in English translation as part of the well known collections of Poincaré's popular writings, but the later versions differed considerably from the first publication. Ewald provides a text of both versions, where deletions are indicated by square brackets, additions by footnotes.

Zermelo's 1905 proof of the well-ordering theorem from the axiom of choice touched off a vigorous debate on the legitimacy of the axiom, including an exchange of letters among Borel, Baire, Hadamard and Lebesgue. Gregory Moore's translation of this exchange is included.

All of Hilbert's published articles on the foundations of mathematics are translated here, with the exception of those available in the van Heijenoort anthology, and two articles of lesser significance. Of particular note is the first English translation of his classic *Axiomatic Thought.* Ewald wittily notes that in spite of Hilbert's fiery polemics against Kronecker, Weyl and Brouwer, the whole debate is a feud among constructivists (p. 1116).

Brouwer himself is represented by some lesser known pieces, including a late article expounding infinitely proceeding sequences. An earlier piece from 1928, *Mathematics, Science, and Language*, describes the mystical creation of the mathematical universe through the "self-unfolding of the intellectual ur-phenomenon." Brouwer's descriptions of this creation are reminiscent of the *Tao Te Ching*; it's a surprise to realize that this lecture stimulated Wittgenstein's return to philosophy.

The second volume includes a very important essay by Ernst Zermelo from 1930 that was unaccountably omitted from van Heijenoort's anthology. Here, Zermelo describes with crystal clarity what were later called "natural models" of set theory, that is, models consisting of the cumulative hierarchy of sets up to a certain level. He concludes that his set-theoretical axioms are not categorical – here this non-categoricity has to be taken in the second order sense, because Zermelo never accepted the formal conception of axiom systems associated with Hilbert and Gödel.

The collection concludes with a Platonist credo of G.H. Hardy, and a manifesto by the young Nicholas Bourbaki. In his introductory note, Ewald does not describe the members of the Bourbaki group, instead continuing the now rather tiresome *canard* of Bourbaki's real existence.

The translations are by various hands, some being revisions of older versions. All the new translations are by the editor, with the exception of the Bolzano selections (translated by Stephen Russ), a piece by Dedekind (David Reed), and the essay by Zermelo (Michael Hallett).

The introductory essays to the selections are all written by the editor, with the exception of the introduction to the Zermelo paper (Michael Hallett). They are very well written and helpful, providing biographical details, pointers to the literature and an analysis of the selections themselves. The approach is in general more rambling and chatty than the austerely scholarly editorial apparatus of van Heijenoort. I learnt all kinds of biographical titbits from Ewald's essays, for example, Johann Lambert's highly personal system of multicoloured dress, and his rules of etiquette which among other things required him to stand at right angles to the person he was addressing (p. 155). In a long footnote (pp. 444-445) to his introductory essay on George Boole, Ewald describes the amazing Boole family. Here we meet the "conscientious bigamist" C.H. Hinton, who married Boole's eldest daughter Mary (as well as one Maude Florence Weldon), wrote on the fourth dimension and invented a gun for firing baseballs, and also Boole's youngest daughter Ethel who wrote the smash bestseller *The Gadfly* and for a time was the mistress of the spy Sydney Reilly, the original of James Bond.

This is an excellent collection, but unfortunately its price puts it beyond the reach of most personal libraries. Any serious institutional collection of philosophy, though, should have this anthology.

Bourbaki has been blamed for following too formal an approach. Indeed, the books are void of much motivation and application, apart from a few of introductions and few chapters on history. Apart from the Bourbaki volumes, there is also a lively Bourbaki seminar which takes place on certain Saturdays in Paris, is open to public, has non-Bourbaki members as invited speakers who present and discuss in advance chosen topics of recent development in mathematics; the expositions are subsequently published. Bourbakiâ€™s notion of structure and the relation to category theory. From Berkeley to Bourbaki. Article. Jun 1999.Â According to the advocates of a "Generalized Darwinism" (GD), the three core Darwinian principles of variation, selection and retention (or inheritance) can be used as a general framework for the development of theories explaining evolutionary processes in the socioeconomic domain. Even though these are originally biological terms, GD argues that they can be re-defined in such a way as to abstract from biological particulars. We argue that this approach does not only risk to misguide positive theory development, but that it may also impede the construction of a coherent evolutionary approach t Nicolas Bourbaki (French pronunciation: [nikÉ”la buÊbaki]) is the collective pseudonym of a group of mathematicians, predominantly French alumni of the Ã‰cole normale supÃ©rieure (ENS). Founded in 1934â€“1935, the Bourbaki group originally intended to prepare a new textbook in analysis. Over time the project became much more ambitious, growing into a large series of textbooks published under the Bourbaki name, meant to treat modern pure mathematics. The series is known collectively as the Ã‰lÃ©ments de Nicolas Bourbaki, 1935â€“???? If you are a mathematician working today, you have almost certainly been influenced by Bourbaki, at least in style and spirit, and perhaps to a greater extent than you realize. But if you are a student, you may never have heard of it, him, them. What or who is, or was, Bourbaki? Check as many as apply. Bourbaki is, or was, as the case may be: the discoverer (or inventor, if you prefer) of the notion of a mathematical structure; one of the great abstractionist movements of the twentieth century