Trivia Quiz

1. The English Civil War was primarily between:
   a) Catholics and protestants;
   b) North and South;
   c) Royalists and Parliamentarians.
2. It happened in the:
   a) 1630's;
   b) 1640's;
   c) 1650's.
3. The Lord Protector was:
   a) James I;
   b) Thomas Cromwell;
   c) Charles I;
   d) Oliver Cromwell;
   e) Charles II.
4. Which of the following events did *not* take place in 1665-7:
   a) The great plague;
   b) The Jacobite rebellion;
   c) Invasion of the Thames by the Dutch Fleet;
   d) The Great Fire of London
5. In the seventeenth century, Berlin was in the state of:
   a) Brandenburg;
   b) Prussia;
   c) Saxony.

The answers

1. The English Civil War was primarily between:
   c) Royalists and Parliamentarians.
2. It happened in the b) 1640's. It started in 1642. There were essentially two rounds of conflict. Charles I was captured and beheaded in 1649.
3. The Lord Protector was d) Oliver Cromwell, from 1651 until his death in 1659.

4. The only one of these events which did *not* take place in 1665-7 was b) The Jacobite rebellion(s) (1715 and 1745).
5. In the seventeenth century Berlin was in the state of a) Brandenburg The state of Prussia was founded in 1701.

Two names are preeminent in the development of calculus:

- Isaac Newton (1642-1727)
- Gottfried Wilhelm Leibniz (1646-1716)

They never met

Isaac Newton

- He was born during the civil war, on a farm in Woolsthorpe in Lincolnshire.
- His father died 2 months before he was born. His mother managed the family farm.
- He had some schooling locally, and went to Cambridge in 1661.
- He studied the works of Descartes, Copernicus, Kepler, Galileo, Wallis and Barrow. (Barrow was the first professor of mathematics at Cambridge.)
- In 1665 when he had just finished his undergraduate studies, the plague started to spread and he returned home for about two years.
- During this time he
  - formulated unified laws of mechanics: *Newton's Laws*
  - developed the theory of calculus, including the fundamental principle that differentiation and integration are inverse principles
  - recognized that white light is the combination of lights of different colours
- He did not promote these theories. When the plague died down he returned to Cambridge to study for a master's degree, was awarded a fellowship at Trinity College, and inherited Barrow's chair in 1669, when Barrow turned to theology.
- Others such as Galileo, Kepler, Hooke, Huygens had ideas on laws of mechanics –on the solar system for example — but Newton's formulation was the most comprehensive
- Fermat, Torricelli, Gregory and Barrow recognised the inverse relationship between differentiation and integration but these were in specific contexts or geometric language or (in the case of Gregory) the work went unnoticed.

Newton's Laws

- *First law of motion* In the absence of external forces, velocity is unchanged.
- *Second law of motion*

$$\mathbf{F} = m\mathbf{a}$$

  where m is the mass of an object, $\mathbf{a}$ is the acceleration vector and $\mathbf{F}$ is the force applied to the object
- *Third law of motion* For every action there is an equal and opposite reaction.

Comments

- The second law is the really important one.The first one can be derived straightforwardly from it (using calculus!) and it implies the existence of reaction forces.
- The first law was also given by Galileo and is known as his *Law of inertia*.
- The second law contrasts with Aristotle's law which could be formulated as

$$\mathbf{F} = m\mathbf{v}$$

  where $\mathbf{v}$ is the velocity vector. Aristotle's may seem to accord more with experience, but does not take into account friction and resistance forces.
- Newton's laws apply not only to single particles and solid bodies, but also large systems of particles in solids, liquids and gases. The ubiquity of second order differential equations in physical problems is due to Newton's second law.
- Newton had a pathological fear of criticism.
- So when he ventured to publish some of his highly original work – on light, for example in 1672 – and attracted criticism from Hooke, Huygens and others – he resolved not to publish any more.
- In the beginning, his only supporter was Isaac Barrow.
- Later, Halley supported him and helped him to publish *Principia*.
- Most of his papers were published long after they were written.

Newton's works on Calculus

- *De Analysi per Aequationes Numero Terminorium Infinitorum* circulated privately in 1669 but published in 1711;
- *Methodus Fluxiorum et Seriorum Infinitorum* written 1671, published 1736;
- *Tractatus de Quadrature Curvarum* written 1676, published 1704.

The material is basically the same in all three works, with some development in notation and ideas

- Infinitesimals of variables occur in *De Analysi.*.
- Infinitesimals of functions – called *fluxions* of *fluents*, occur in *Methodus Fluxiorum.*.
- Newton was extremely fond of series and used them extensively - in integration, for example, for functions such as $1/(1+x^2)$.

Principia

- *Philosophiae Naturalis Principia Mathematica* was first published in 1687.
- There were a number of improvements in the second edition in 1713. The third edition appeared in 1726.
- *Principia* has three books:
  - The first book is about the *two body problem* (one planet around a sun)
  - The second book is about hydrodynamics, including the design of ships to minimise the resistance to motion: possibly the earliest such treatment.
  - The third book is about the solar system.
- The collaboration between Halley and Newton centred round the two-body problem.
- The *inverse square law* says that gravitational force is proportional to the inverse square of the distance between two bodies, that is,

$$\mathbf{r}'' = -a\frac{\mathbf{r}}{|\mathbf{r}|^3}.$$

- Hooke and Newton had some correspondence about the inverse square law. Hooke proposed it as a law to Newton, who then calculated that the law gave rise to elliptical orbits of a single planet round a sun (but did not tell Hooke this).
- Later Halley heard about the inverse square law from Hooke and asked Newton for help in calculating motion.
- Newton showed Halley his calculations.
- Halley persuaded him to publish and assisted him as editor.

Here is a segment of Book 2 of Principia (translated into English)

Note the lack of notation and also the geometric language. More analytical arguments are given by Newton in other papers.

- In 1685 James II came to the throne and Newton became involved in politics, first within the university and then beyond – becoming one of Cambridge University's two members of parliament.
- James II was overthrown and William and Mary were invited to the throne.
- In 1693, Newton had a nervous breakdown. In 1696 accepted a position as Master of the Mint, and eventually gave up his chair at Cambridge.
- He was elected president of the Royal Society in 1703, and held this position untill his death.

Elliptical orbits

The derivation of elliptical orbits from the equation

$$\mathbf{r}'' = -a\frac{\mathbf{r}}{|\mathbf{r}|^3}. \tag{1}$$

is as follows.

- We use polar coordinates (r, θ) for the position of the planet relative to the sun, taking the sun at $(0, 0)$, so that $r = |\mathbf{r}|$
- We write $\widehat{\mathbf{r}}$ and $\widehat{\theta}$ for the unit vectors in the radial and tangential directions, so that $\mathbf{r} = r\widehat{\mathbf{r}}$.
- It is standard that
$$\frac{d}{dt}\widehat{\mathbf{r}} = \theta'\widehat{\theta}, \quad \frac{d}{dt}\widehat{\theta} = -\theta'\widehat{\mathbf{r}}$$
.

Using this, (1) becomes

$$r'' - r\theta'^2 = -\frac{a}{r^2} \tag{2}$$

$$2r'\theta' + r\theta'' = 0 \tag{3}$$

Now multiplying (3) by r gives

$$\frac{d}{dt}(r^2\theta) = 0$$

and hence

$$r^2\theta' = b$$

for a constant b. this is *Kepler's Law*, that area is swept out at a constant rate. Substituting for θ' in (2) gives

$$r'' - \frac{b^2}{r^3} + \frac{a}{r^2} = 0 \tag{4}$$

Multiplying (4) by r' gives

$$\frac{d}{dt}\left(r'^2 + \frac{b^2}{r^2} - \frac{2a}{r}\right) = 0 \tag{5}$$

and hence

$$r'^2 = C + \frac{2a}{r} - \frac{b^2}{r^2} = c^2 - \left(\frac{b}{r} - a\right)^2 \tag{6}$$

for constants C and c Now writing

$$r' = \frac{dr}{dt} = \theta' \frac{dr}{d\theta} = \frac{b}{r^2}\frac{dr}{d\theta}$$

we have

$$\frac{b}{r^2\sqrt{c^2 - (b/r - a)^2}}\frac{dr}{d\theta} = 1 \tag{7}$$

So

$$\int \frac{bdr}{r^2\sqrt{c^2 - (b/r - a)^2}} = \int d\theta \tag{8}$$

Making the change of variable $c\cos u = b/r - a$ yields $c\sin u\, du = bdr/r^2$ and

$$\int du = \int d\theta, \tag{9}$$

and so $u = \theta + \alpha$ for some α and

$$b = r(a + c\cos(\theta + \alpha)) \tag{10}$$

for constants a, b, c and α. This is the equation of an ellipse with focus at the origin.

references

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3000 precious jewels (contain) in the Imperial State Crown. In 1671 a daring attempt (make) to steal the Crown Jewels by a man named Captain Blood.Â The Tower of London was built by William the Conqueror in 1078 as a castle and palace. Since that time it has been expanded to its present size, and has been used as an armoury, a zoo, a royal mint, a prison, and a museum. At the time when it was a prison a lot of people were locked in the Tower for their religious beliefs or suspected treason.Â Some of the prisoners were allowed to walk in the grounds, live in comfortable rooms and receive visitors. Many convicted were publicly executed on Tower Hill. They were beheaded with the block and ax, which are kept and shown in the Tower Armoury now. The Great Fire of London happened in Central London in 1666. The fire lasted for just under five days, from 2 September until 6 September. It is believed to have destroyed the homes of 70,000 of the 80,000 people that lived in the city. It is not known how many people died in the fire. Only a few deaths are certain, but for many of the victims there were no records. Also, the fire may have cremated many, leaving no recognisable remains. It threatened the aristocratic district of Westminster (the The London of the middle of the 17th century was a city of narrow, dirty streets. Indeed, the streets were so narrow that it was often possible for a person at a window on one side of the street to shake hands with a neighbour on the otherside. There was little light and air. Rubbish lay piled up in dark corners. It is no wonder that epidemics were common.Â The streets were empty, shops were closed and there were few boats on the Thames. Every house in which there were sick people was shut up, and no one was allowed to go in or out, and the door of the house was marked with a red cross. The following year the Great Fire took place. It broke out late on a Saturday night in a street not far from London Bridge. The summer had been dry, a hot east wind blew and the fire spread quickly. Jacobite, in British history, a supporter of the deposed Stuart king James II (Latin: Jacobus) and his descendants. Support for â€˜the king over the water,â€™ as the exiled claimants to the throne were known, retained a sentimental appeal after the movementâ€™s decline, especially in the Scottish Highlands.Â Assistant Master and Professor of History, Selwyn College, University of Cambridge. Consultant editor for the Oxford Dictionary of National Biography . See Article History. Jacobite , in British history, a supporter of the exiled Stuart king James II (Latin: Jacobus) and his descendants after the Glorious Revolution . The political importance of the Jacobite movement extended from 1688 until at least the 1750s.